Non linear dynamics of the glottis :

Phase Portraits and Lyapunov Exponents

Maurice Ouaknine

Laboratoire d'Audio-Phonologie Expérimentale et Clinique
Université de la Méditerranée
Marseille (France)

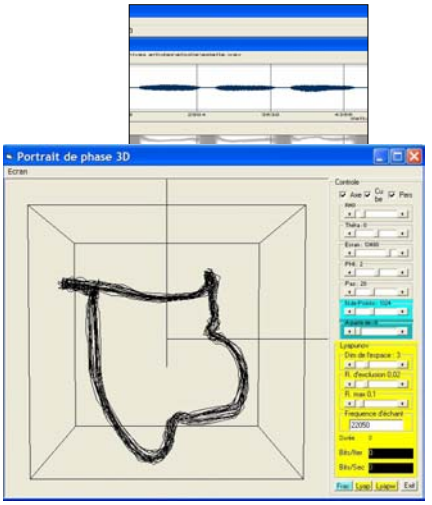
Signal Analysis

Tools available

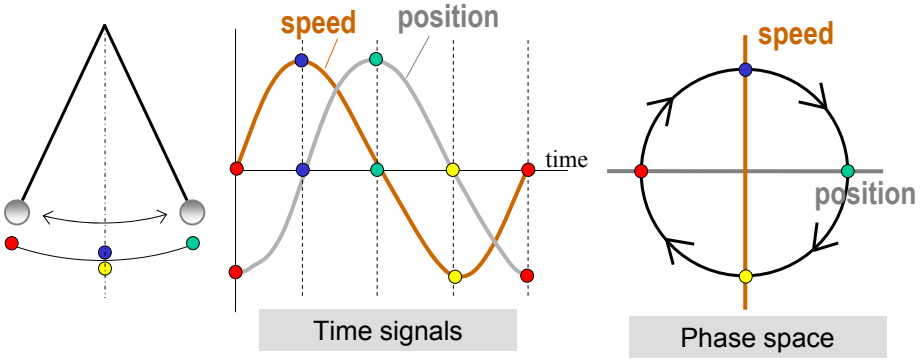
- Wave display
- Spectrogram
- Jitter

- Cepstrum
- Autocorrelation

- Phase portraits



Construction of a Phase Portrait : Pendulum

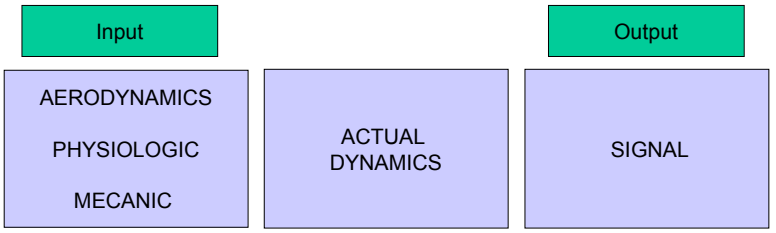


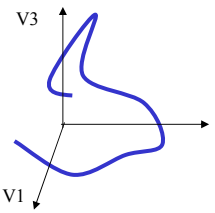
Time signals

Phase space

- Knowledge of the position and the speed of the pendulum is sufficient to characterize its dynamics completely
- Succession of the states of a periodic system describes a closed curve in the phase space

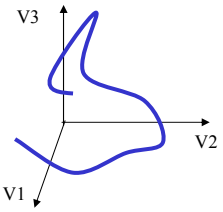
Phase Portrait : representation of the dynamics of a system





V1,2,3,n : Actual parameters

Construction of a Phase Portrait : GLOTTIS

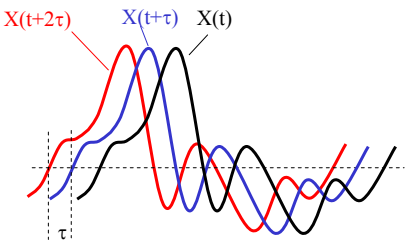


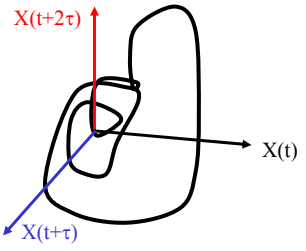
V1,2,3 : Unknown actual parameters

TIME DELAY METHOD
D Ruelle, F Takens

Input		Output
AERODYNAMICS PHYSIOLOGIC MECANIC	?	VOCAL SIGNAL

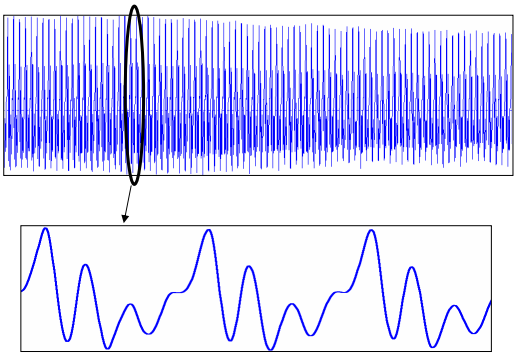
Construction of a Phase Portrait : time delay method

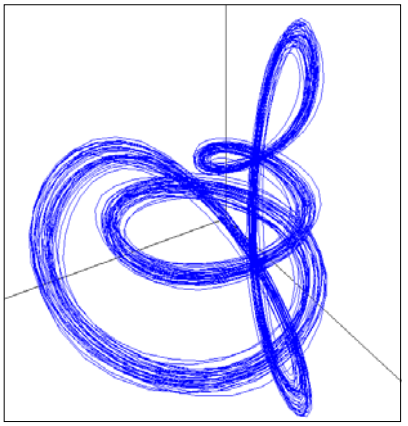




Example 1 : normal voice

Sample of sustained Vowel/a/

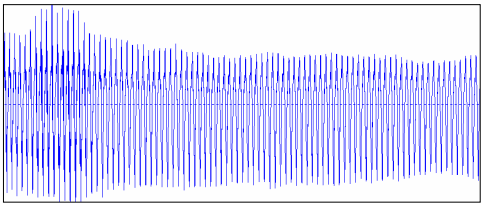


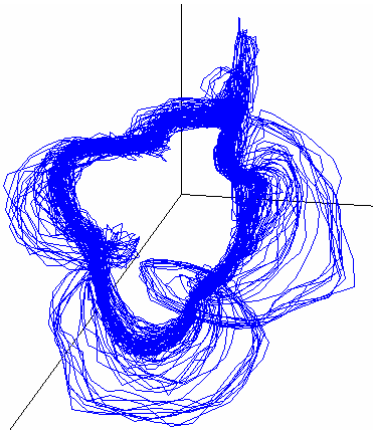


-Quasi-periodic attractor

-Closed-loop trajectories

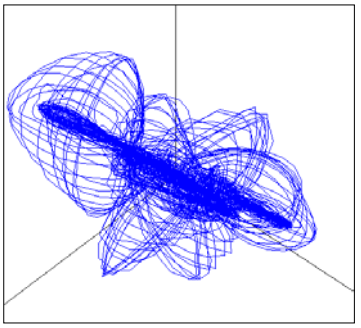
Example 2 : severe dysphonia

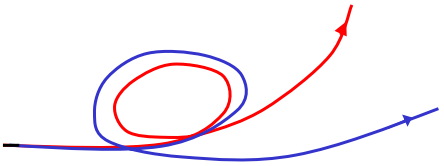




Quantification of a Phase Portrait

- Fractal dimensions
  - Number of degrees of freedom
- Lyapunov Exponent
  - Divergence of initially close trajectories
    - Divergence : Sensitivity to Initial Conditions (SIC)





Determination of Lyapunov exponent

After n iterations, error amplification factor would be:

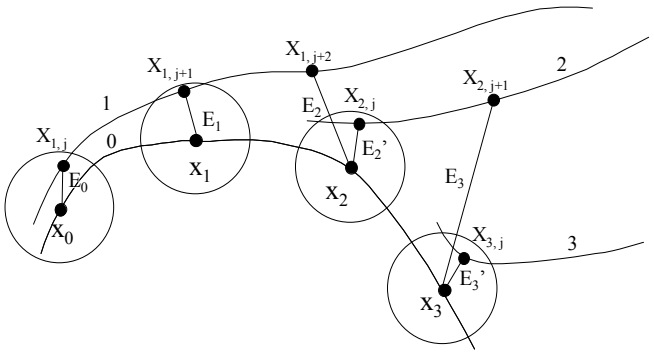
$$\left|\frac{E_n}{E_0}\right| = \left|\frac{E_n}{E_{n-1}}\right|\left|\frac{E_{n-1}}{E_{n-2}}\right|\ldots\left|\frac{E_1}{E_0}\right|$$

Lyapunov coefficient characterizes the logarithm of increase in relative error during iteration

$$\lambda = \frac{1}{n}\sum_{k=1}^{n}\log\left|\frac{E_k}{E_{k-1}}\right| \quad for \quad n\to\infty \;; E_0 \to 0$$

Giovanni A, Ouaknine M, Triglia JM. Determination of largest Lyapunov Exponents of Vocal Signal: application to unilateral laryngeal paralysis. J Voice, 1999;13:341-354

Choice of neighbor points of the reference trajectory



In an experimental series, the main problem is selection of neighboring points in the reference orbit.

Signal instability : clinical measurements